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Name

1. Suppose that the random variables X and Y have the joint pdf

$$f_{XY}(x,y) = \begin{cases} xe^{-x(y+1)} & x > 0, \ y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) *(10 points)* Find the pdf of X.

(b) *(10 points)* Find the pdf of Y.

$$f_{XY}(x, y) = \begin{cases} xe^{-x(y+1)} & x > 0, \ y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

(c) *(10 points)* Find the pdf of $Y|X$ (i.e., Y given X).

(d) *(10 points)* Find the pdf of XY.

$$f_{XY}(x,y) = \begin{cases} xe^{-x(y+1)} & x > 0, \ y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

(e) *(10 points)* Find the pdf of $\max(X, Y)$.

(f) *(10 points)* Find $F_X(x)$.

$$f_{XY}(x,y) = \begin{cases} xe^{-x(y+1)} & x > 0, \ y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

(g) *(10 points)* Find $F_{XY}(\frac{1}{2}, \frac{3}{4})$.

(h) *(5 points)* Find $F_X(\frac{1}{2})$.

(i) *(5 points)* Are X and Y independent? Why?

2. Let X be the outcome of rolling one fair die, and Y be equal to 1 if a fair coin is flipped and results in heads, and 0 otherwise.

   (a) *(10 points)* What is the joint pdf of X and Y?

   (b) *(10 points)* Find the pdf of $X + Y$.

3. **Extra Credit:** No proofs are needed for the following questions. If you don't know the answer, feel free to make an educated guess. You can give the pdf/pmf, or just give the name and parameters of the distribution.

(a) *(2 points)* If $X_1, \ldots, X_n \overset{iid}{\sim} P(\lambda)$, what is the distribution of $Y = \sum_{i=1}^{n} X_i$?

(b) *(2 points)* If $X_1, \ldots, X_n \overset{iid}{\sim} \text{Exp}(\lambda)$, what is the distribution of $Y = \sum_{i=1}^{n} X_i$?

(c) *(2 points)* If $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, what is the distribution of $Y = \sum_{i=1}^{n} X_i$?

(d) *(2 points)* If $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, what is the distribution of $Y = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma} \right)^2$?

(e) *(1 point)* What is the name and parameter value of the marginal distribution of Y in question 2?

(f) *(1 point)* What is the name and parameter value of the distribution of $Y|X$ in question 1(c)?