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Name

For *any* of these questions, you can use *any* result from my cheat sheet = Appendix C.

1. *(20 points)* To reward herself for finishing STAT 394/395, Stephanie gets herself a unicorn. However, the unicorn escapes and she must find it. Assume that Stephanie and her unicorn are each at different points X and Y that are uniformly distributed on a road of length L. If X and Y are independent, find the expected value and variance of the distance between Stephanie and her unicorn.

2. *(10 points)* Let $X_1 \sim \mathrm{N}(4, \sigma = 2)$ and $X_2 \sim \mathrm{N}(3, \sigma = 5)$ be independent from each other. Determine the distribution of $Y = 2X_1 - 3X_2 + 6$, and prove your answer.

3. Suppose that X and Y are two random variables jointly distributed with pdf

$$f_{X,Y}(x,y) = \begin{cases} y^2 e^{-y(x+1)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) *(10 points)* Determine $f_{X|Y}(x|y)$.

(b) *(5 points)* What is the name and parameter value of the distribution found in part (a) above?

4. Let $f_X(x) = \lambda e^{-\lambda(x-\alpha)},\ x > \alpha$ for some $\lambda > 0$.

   (a) *(15 points)* Find the moment generating function for this distribution. For what range of t does this exist?

(b) *(15 points)* Use this moment generating function to determine $\mathrm{E}[X]$ and $\mathrm{Var}(X)$.

5. *(15 points)* When a sample of $2n + 1$ random variables are observed, the $(n + 1)^{\text{th}}$ smallest value is called the *sample median*. Suppose a random sample of size 3 is taken from a uniform distribution over (0,1). Find the probability that the sample median is between $\frac{1}{4}$ and $\frac{3}{4}$.

6. *(10 points)* From past experience, Nate knows that the test score of a student taking his final exam is a random variable with mean of 75 and standard deviation of 5. How many students would have to take the exam so as to ensure, with probability at least 90%, that the class average would be within 5 of 75? Use the weak law of large numbers.

7. **Extra Credit Problems**

(a) *(5 points)* In problem 7, *without using the standard deviation*, give an upper bound for the probability that a student's test score will be 85 or above. That is, we are assuming that the standard deviation is unknown.

(b) *(5 points)* Now using the standard deviation if necessary, give a lower bound of the probability that a student will score between 65 and 85.