STAT/MATH 394 A **Midterm**
Derby **July 8, 2009**

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Name

1. An experiment consists of both tossing a fair coin and rolling a fair die.

   (a) *(2 points)* Construct a sample space for this experiment.

   List the elements in the following events:

   (b) *(2 points)* Heads and an odd number on the die.

   (c) *(3 points)* Tails or an even number on the die.

   (d) *(3 points)* Neither heads nor a six on the die.

2. Let the pmf for the random variable X be given by

$$p_X(x) = \frac{\binom{10}{x}\binom{90}{20-x}}{\binom{100}{20}}, \quad x \in \{0, 1, 2, \ldots, 10\}.$$

Find the following:

(a) *(5 points)* $\mathrm{P}(X \leq 2)$

(b) *(5 points)* $F_X(2)$, the distribution function at $x = 2$.

$$p_X(x) = \frac{\binom{10}{x}\binom{90}{20-x}}{\binom{100}{20}}, \quad x \in \{0, 1, 2, \ldots, 10\}.$$

(c) *(5 points)* $F_X(2.5)$.

(d) *(5 points)* $\mathrm{P}(X \geq 2)$.

3. Let A and B be events in a sample space S with $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$, and $\mathrm{P}(A \cap B^c) = 0.3$. Compute the following:

   (a) *(5 points)* $\mathrm{P}(A \cap B)$.

   (b) *(5 points)* $\mathrm{P}(A \cup B)$.

   (c) *(5 points)* $\mathrm{P}(A \cup B^c)$.

4. Consider the set of digits $\{1, 3, 4, 5, 7, 8, 9\}$.

   (a) *(5 points)* If digits cannot be repeated, how many different four-digit numbers can be formed from this set of digits?

   (b) *(5 points)* If digits cannot be repeated and the middle digit must be an odd digit, how many different three-digit numbers can be formed from this set of digits?

   (c) *(5 points)* If digits cannot be repeated, how many different three-digit numbers less than 700 can be formed from this set of digits?

5. I pick three cards from a standard 52-card deck. What is the probability of getting

   (a) *(5 points)* A triple (three cards of the same number, but of different suits)?

   (b) *(5 points)* A flush (three cards of the same suit, but of different numbers)?

(c) *(5 points)* All different numbers?

(d) *(5 points)* A pair (two cards of the same number), plus a singleton that is one higher than the value of that pair? Note that an Ace can be high or low – we can consider an Ace to be one lower than 2, but one higher than King.

6. Suppose that the amount of money (in dollars) that a typical STAT/MATH 394 undergrad has saved is a random variable X with a distribution function given by

$$F_X(x) = \begin{cases} \frac{1}{2}e^{-(x/50)^2} & x \leq 0 \\ 1 - \frac{1}{2}e^{-(x/50)^2} & x > 0 \end{cases}.$$

Note that a negative amount of savings represents a debt. Also, assume that X is a continuous random variable (even though strictly speaking, it isn't).

(a) *(5 points)* Determine the pdf $f_X(\cdot)$.

(b) *(5 points)* What is the probability that the amount of savings by a 394 student will be more than \$50?

(c) *(5 points)* What is the probability that the amount of savings by a 394 student will be between -\$50 and \$50, noninclusive (not including the endpoints)?

(d) *(5 points)* What is the probability that the amount of savings by a 394 student will be equal to \$50?

7. **Extra Credit:**

(a) *(2 points)* For problem 1(a), construct a non-minimal sample space for this experiment. (If your answer to problem 1(a) is already a non-minimal sample space, just give the same answer here)

(b) *(4 points)* Suppose I roll 10 fair dice, and X is the number of sixes that I have in this roll. What is the pmf for X? (No proof is necessary – just give me the formula)

(c) *(4 points)* Come up with a context for a random variable X which would have the pmf $p_X(\cdot)$ given in problem 2. An example of a context is "I roll 10 dice, and X is the number of sixes that I have in this roll." Note that this example is the wrong context for the pmf in problem 2.