
Name

1. Dreaming of riches and lured by the number of foreclosures in the Seattle area, Vong Ly decides to buy 5 condos as an investment. He assumes that the probability he will make a profit on each within five years is 0.7. Assuming independence of the condos,

(a) (*4 points*) What is the probability that he makes a profit on every one of them within five years?

(b) (*5 points*) What is the probability he makes a profit on 5 of them, given he makes a profit on at least one of them?

(c) (*5 points*) What are the expected value and standard deviation of the number of condos Vong expects to be profitable over five years?

2. Let the random variable X be defined by

$$p_X(x) = \begin{cases} \frac{1}{10} & x \in \{-3, -2, -1, 0\} \\ \frac{1}{15} & x \in \{1, 2, 4\} \\ \frac{2}{15} & x \in \{3, 5, 6\} \\ 0 & \text{otherwise} \end{cases} .$$

(a) (2 points) Is this a valid pmf? (What axioms should this fulfill? Just the first and second are fine)

(b) (5 points) Find the pmf for $Y = X^2$.

(c) (5 points) Find the expected value and variance of X .

(d) (4 points) Find the expected value and variance of $Z = 4X - 3$.

3. A box contains 3 red balls and 2 green balls. Mario, Luigi, Princess Peach and Donkey Kong randomly draw balls from the box, one ball per person, *without replacement* of balls between drawings. Each person who draws a red ball gets 5 mushrooms, and each person who draws a green ball gets 10 mushrooms. The princess draws first, then Mario, then Luigi, then Donkey Kong.

(a) (10 points) Let X be the random variable denoting the amount of mushrooms received by the second person (Mario). Find $E[X]$.

(b) (10 points) Let T be the random variable denoting the total amount of mushrooms received by all four people, combined. Find $\text{Var}(T)$.

4. On the planet of the Yip Yips, the natives play poker with a standard deck of cards, but each player has 9 cards in his/her hand. Find the probability that a Yip Yip hand will contain the following:

(a) (10 points) Three pairs and a triple.

(b) (10 points) Two triples and three different singletons.

(c) (10 points) All 9 cards of the same suit (e.g., all clubs, all diamonds, etc).

(d) (10 points) A sequence (e.g., {A, 2, 3, 4, 5, 6, 7, 8, 9} or {2, 3, 4, 5, 6, 7, 8, 9, 10}, etc), where the ace may be either high or low.

5. (10 points) During the STAT 394 A midterm, Nate walks out of the room for a few minutes, and later on suspects that some of the 46 students cheated while he was out of the room. Each student is given a polygraph test. From past experience, it is known that the polygraph is 90% reliable when administered to a guilty suspect and 98% reliable when given to an innocent suspect. Suppose that of the 46 students who took the midterm, only 6 actually cheated. What is the probability that a given suspect is innocent, given that the polygraph says that he or she is guilty?