STAT/MATH 394 A **Homework 4**
Derby **Due July 22, 2009 at 8:30 am**

This entre homework is for extra credit.

10 cards are drawn from a standard 52-card deck.

1. **Extra Credit:** *(10 points)* What is the probability that we have a sequence of just 8 numbers (i.e., it's not part of a sequence of 9 or 10 numbers)?

2. **Extra Credit:** *(2 points)* What is wrong with this solution to problem 1 above?:

   Note that for the values, we have 7 possible sequences:

$$\begin{array}{c} A,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8 \\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9 \\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10 \\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ J \\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ J,\ Q \\ 6,\ 7,\ 8,\ 9,\ 10,\ J,\ Q,\ K \\ 7,\ 8,\ 9,\ 10,\ J,\ Q,\ K,\ A \end{array}$$

   Since we want sequences of *only* 8 values, we must be sure to avoid forming a 9-value sequence when we add our 2 extra numbers. For the first and last of the above sequences, we have 12 possible values for the other 2 cards (e.g., for the first sequence, we can use all values except for a 9). For the other 5 sequences, we have 11 possible values for the other 2 cards (e.g., for the second sequence, we can use all values except for A or 10). Therefore,

$$\frac{\overbrace{2}^{\text{first or last sequence}} \cdot \overbrace{\binom{12}{1}^2}^{\text{choose 2 extra values}} \cdot \overbrace{4^{10}}^{\text{choose suits for 10 values}} + \overbrace{5}^{\text{remaining 5 sequences}} \cdot \overbrace{\binom{11}{1}^2}^{\text{choose 2 extra values}} \cdot \overbrace{4^{10}}^{\text{choose suits for 10 values}}}{\binom{52}{10}} = \frac{4^{10} \cdot (2 \cdot 12^2 + 5 \cdot 11^2)}{\binom{52}{10}}$$

$$\approx 5.92\%.$$

3. **Extra Credit:** *(2 points)* What is wrong with this solution to problem 1 above?:

   There are 7 ways to make a sequence of 8 cards.

A	2	3	4	5	6	7	8	(set cannot include 9)
2	3	4	5	6	7	8	9	(set cannot include A or 10)
3	4	5	6	7	8	9	10	(set cannot include 2 or J)
4	5	6	7	8	9	10	J	(set cannot include 3 or Q)
5	6	7	8	9	10	J	Q	(set cannot include 4 or K)
6	7	8	9	10	J	Q	K	(set cannot include 5 or A)
7	8	9	10	J	Q	K	A	(set cannot include 6)

$$\frac{\left(2 \times 4^8 \times \binom{40}{2}\right) + \left(5 \times 4^8 \times \binom{36}{2}\right)}{\binom{52}{10}} \approx 1.95\%$$

   $(2 \times 4^8 \times \binom{40}{2})$ gives the number of combinations of 10 cards that form a sequence that begins or ends with an ace ($\times 2$ because there are two of them). 4^8 gives the number of different ways you can permute the suits within a given sequence. $\binom{40}{2}$ gives the number of different ways to choose the last two cards that are not part of the sequence. Why 40? In the sequences capped by an ace, there is only one face value (and therefore a total of 4 cards, one for each suit) that cannot be included in the set. Therefore, $52 - 8$ (cards already chosen) -4 (cards excluded from set) $= 40$ (possible cards to choose the last two cards from).

   $(5 \times 4^8 \times \binom{36}{2})$ gives the number of combinations of 10 cards that form a sequence that doesn't begin or end with an ace ($\times 5$ because there are 5 of them). 4^8 gives the number of different ways you can permute the suits within a given sequence. $\binom{36}{2}$ gives the number of different ways to choose the last two cards that are not part of the sequence. Why 36? In the sequences not capped by an ace, there are two face values (and therefore a total of 8 cards) that cannot be included in the set. Therefore, $52 - 8$ (cards already chosen) -8 (cards excluded from set) $= 36$ (possible cards to choose the last two cards from).

4. **Extra Credit:** *(2 points)* What is wrong with this solution to problem 1 above?:

   So first we can simply list out how many 8-card sequences are possible. First we have A-8, then 2-9, all the way up to 7-A. Therefore there are 7 different 8-card straights if we ignore the suit. However, now we must assign each of these cards a suit. Finally we determine the last 2 cards in any given hand. If the straights found contain an Ace, then there is only one card that would increase the straight, the other four cards are safe. For all other straights, there are two cards that are not safe and three that are. Therefore, our probability is

$$\frac{\left[2 \cdot \binom{4}{2} + 5 \cdot \binom{3}{2}\right] \cdot \binom{4}{1}^{10}}{\binom{52}{10}} \approx 0.18\%.$$

5. **Extra Credit:** *(2 points)* What is wrong with this solution to problem 1 above, which is a variation of the last answer above?:

$$\frac{\left[2\cdot\binom{12}{2}+5\cdot\binom{11}{2}\right]\cdot\binom{4}{1}^{10}}{\binom{52}{10}}\approx 2.70\%.$$

6. **Extra Credit:** *(2 points)* What is wrong with this solution to problem 1 above?:

   For P(1 → 8) and P(7 → 14), there is only one number that we can't have, while for P(2 → 9), P(3 → 10), ..., P(6 → 13), there are two that we can't have. Let

$$a_1 = \underbrace{4^8}_{\substack{\text{choose}\\\text{suits}\\\text{for 8}\\\text{values}}}\left[\underbrace{\binom{8}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from 8}}}\cdot\underbrace{\binom{3}{2}}_{\substack{\text{choose}\\\text{2 suits}\\\text{for them}}}+\underbrace{\binom{8}{2}}_{\substack{\text{choose}\\\text{2 values}\\\text{from 8}}}\cdot\underbrace{\binom{3}{1}^2}_{\substack{\text{choose}\\\text{1 suit}\\\text{for each}}}+\underbrace{\binom{8}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from 8}}}\cdot\underbrace{\binom{3}{1}}_{\substack{\text{choose}\\\text{1 suit}\\\text{for it}}}\cdot\underbrace{\binom{4}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from}\\\text{other 4}}}\cdot\underbrace{\binom{4}{1}}_{\substack{\text{choose}\\\text{1 suit}\\\text{for it}}}+\underbrace{\binom{4}{2}}_{\substack{\text{choose}\\\text{2 values}\\\text{from}\\\text{other 4}}}\cdot\underbrace{\binom{4}{1}^2}_{\substack{\text{choose}\\\text{1 suit}\\\text{for each}}}+\underbrace{\binom{4}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from}\\\text{other 4}}}\cdot\underbrace{\binom{4}{2}}_{\substack{\text{choose}\\\text{2 suits}\\\text{for each}}}\right]$$

   and

$$a_2 = \underbrace{4^8}_{\substack{\text{choose}\\\text{suits}\\\text{for 8}\\\text{values}}}\left[\underbrace{\binom{8}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from 8}}}\cdot\underbrace{\binom{3}{2}}_{\substack{\text{choose}\\\text{2 suits}\\\text{for them}}}+\underbrace{\binom{8}{2}}_{\substack{\text{choose}\\\text{2 values}\\\text{from 8}}}\cdot\underbrace{\binom{3}{1}^2}_{\substack{\text{choose}\\\text{1 suit}\\\text{for each}}}+\underbrace{\binom{8}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from 8}}}\cdot\underbrace{\binom{3}{1}}_{\substack{\text{choose}\\\text{1 suit}\\\text{for it}}}\cdot\underbrace{\binom{3}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from}\\\text{other 3}}}\cdot\underbrace{\binom{4}{1}}_{\substack{\text{choose}\\\text{1 suit}\\\text{for it}}}+\underbrace{\binom{3}{2}}_{\substack{\text{choose}\\\text{2 values}\\\text{from}\\\text{other 3}}}\cdot\underbrace{\binom{4}{1}^2}_{\substack{\text{choose}\\\text{1 suit}\\\text{for each}}}+\underbrace{\binom{3}{1}}_{\substack{\text{choose}\\\text{1 value}\\\text{from}\\\text{other 3}}}\cdot\underbrace{\binom{4}{2}}_{\substack{\text{choose}\\\text{2 suits}\\\text{for each}}}\right]$$

   so that our answer is

$$\frac{2\cdot a_1+5\cdot a_2}{\binom{52}{10}}\approx 1.95\%.$$

7. **Extra Credit:** *(2 points)* Modifying your answer from problem 1, what is the probability that we have a sequence of just 8 numbers (i.e., it's not part of a sequence of 9 or 10 numbers), all of the same suit?