Do either problem 7 or problem 11 – only one will count for credit. If both problems are done, credit will be taken from the one that received the most points.

1. Assume (X, Y) is a two-dimensional continuous random variable with density

$$f_{X,Y}(x, y) = \frac{1}{x}, \quad 0 < y < x < 1.$$

   (a) *(6 points)* Find the marginal densities for X and Y.
   (b) *(1 point)* What is the name of the marginal distribution of X?
   (c) *(2 points)* Are X and Y independent?
   (d) *(3 points)* Find $F_X(\frac{1}{4})$.
   (e) *(3 points)* Find $F_{X,Y}(\frac{1}{2}, \frac{1}{4})$.
   (f) *(3 points)* Find $F_{X,Y}(-\frac{1}{2}, -\frac{1}{4})$.

2. An urn contains four balls colored red, yellow, blue and green. Let A be the event that the red or green ball is selected, let B be the event that the yellow or green ball is selected, and let C be the event that the blue or green ball is selected.

   (a) *(3 points)* Are A, B and C pairwise independent?
   (b) *(3 points)* Are A, B and C independent?
   (c) *(3 points)* Are A and BC independent?

3. Suppose

$$f_{X,Y}(x, y) = ke^{-(x+y)}, \quad 0 \le x < \infty, \quad 0 \le y < \infty.$$

   (a) *(3 points)* What is the value of k?
   (b) *(2 points)* Are X and Y independent? Why or why not?
   (c) *(3 points)* Determine $F_Y(3)$.
   (d) *(3 points)* Determine $F_{X,Y}(5, 3)$.
   (e) *(3 points)* Find the marginal pdf of X. What is the name of this distribution?

4. Suppose

$$f_{X,Y}(x, y) = ke^{-(x+y)}, \quad 0 \le x \le y < \infty.$$

   (a) *(3 points)* What is the value of k?
   (b) *(2 points)* Are X and Y independent? Why or why not?
   (c) *(3 points)* Determine $F_Y(3)$.
   (d) *(3 points)* Determine $F_{X,Y}(5, 3)$.

5. *(4 points)* For one jury of three judges, two of them have a probability p of making the correct decision, while the third simply flips a fair coin for each decision. The majority of these three decisions rules. Another jury of just one man has the probability p of making the correct decision. Which jury has the better probability of making the correct decision?

6. An urn contains 12 balls, of which 4 are white. Five balls are drawn and laid aside (not replaced in the urn), their color unnoted.

    (a) *(4 points)* Another ball is drawn. What is the probability that it will be white?

    (b) *(4 points)* A sample of size 2 is drawn. What is the probability that it will contain exactly one white ball?

    (c) *(2 points)* What is the conditional probability that the sample of size 2 will contain exactly 2 white balls, given that it contains at least 1 white ball?

7. On a STAT/MATH 394 quiz show, three boxes each contain 25 envelopes. In the first box, 24 of the envelopes each contain $100; in the second box, 15 contain $100; and in the third,, 12 contain $100. The rest of the envelopes are empty. Stephanie rolls a fair die and selects an envelope at random from the first box if she rolls a 1; from the second box if she rolls a 2 or 3; and from the third box if she rolls a 4,5 or 6.

    (a) *(4 points)* What is the probability that she wins $100?

    (b) *(4 points)* What is the probability that she selected an envelope from the first box, given that she won $100?

    (c) *(4 points)* Given that she did not win $100, what is the probability that she selected an envelope from the third box?

8. The length of time before a concrete block of sidewalk poured by the city of Seattle begins to crumble is approximately normally distributed with $\mu = 15$ years and $\sigma = 4$ years.

    (a) *(3 points)* What is the probability that a block selected at random from those poured by this company starts to crumble within 12 years of being poured?

    (b) *(3 points)* What is the probability that a block selected at random from those poured by this company does not start to crumble for at least 18 years?

    (c) *(3 points)* What is the probability that one of its blocks will begin crumbling between 10 and 15 years after it is poured?

    (d) *(3 points)* The company can control the mean without changing the standard deviation by varying the amount of sand in the concrete. What mean should it use so that the probability that one of its blocks begins to crumble within 7 years is 0.063?

9. Oompa Loompas in Willy Wonka's chocolate factory incur accidents following a Poisson process at the rate of 2 accidents per week.

   (a) *(3 points)* What is the probability that there will be 2 or fewer accidents in one week?

   (b) *(3 points)* What is the probability that there will be 2 or fewer accidents in two weeks?

   (c) *(3 points)* What is the probability that there will be 2 or fewer accidents in each of 2 consecutive weeks?

   (d) *(3 points)* What is the probability that there will be 4 or fewer accidents in 2 weeks?

   (e) *(2 points)* Are the answers to the above two questions the same or different? Why?

   (f) *(3 points)* Given that an accident just happened, what is the probability that one week will pass before the next accident?

   (g) *(3 points)* Given that the last accident happened a week ago, what is the probability that one week will pass before the next accident?

   (h) *(2 points)* Are the answers to the above two questions the same or different? Why?

   (i) *(3 points)* Let X denote the time between accidents. What is the pmf/pdf of X? Is this a pmf or pdf?

   (j) *(3 points)* What is the distribution function of X as defined above? What is the name of it?

   (k) *(3 points)* What is the probability that exactly 5 days, 10 hours, 47 minutes and 22 seconds will pass between accidents?

10. **Extra Credit:** *(4 points)* A fair die is tossed 12 times. What is the probability that we obtain two of each value?

11. An urn contains 14 rubies and 10 diamonds; a second urn contains 10 rubies and 2 diamonds. A fair die is rolled. If the outcome of the roll of the die is 1, you may select a gem at random from the first urn. Otherwise, you select a gem at random from the second urn.

   (a) *(4 points)* What is the probability of selecting a diamond?

   (b) *(4 points)* What is the probability of rolling a 1 and selecting a diamond?

   (c) *(4 points)* What is the probability of rolling a 1, given that a diamond was selected?