STAT/MATH 394 A **Revised Homework 2**
Derby **Due July 10, 2009 at 8:30 am**

1. Suppose that the duration in minutes of long-distance telephone calls made from Seattle is found to be a random phenomenon, with a distribution function given by

$$F_X(x) = 1 - \frac{1}{2}e^{-x/3} - \frac{1}{2}e^{-\lfloor x/3 \rfloor}, \quad x \geq 0,$$

in which the expression $\lfloor y \rfloor$ is the largest integer less than or equal to y (i.e., the *floor* of y).

   (a) *(2 points)* Sketch this distribution function.
   (b) *(2 points)* Is X a discrete or continuous random variable?
   (c) What is the probability that the duration in minutes of a long-distance phone call is
      i. *(4 points)* more than 6 minutes?
      ii. *(4 points)* not less than 6 minutes?
      iii. *(4 points)* less than 4 minutes?

2. *(5 points)* In Example 2.6.1 on page 20 of the notes, we see that at the T-Centralen train station, **in each direction** trains depart every ten minutes, and that since passengers arriving at this station don't know the exact schedule, **in each direction** trains arrive uniformly over a ten-minute interval.

   Now consider the case of Cory, who finishes his daily homework at random times between 3 and 5pm. After he finishes his homework, he goes drinking with either his electrical engineering buddies at a bar uptown, or with his math buddies at a bar downtown. He takes the first subway that comes in either direction and drinks with the one he is first delivered to. His electrical engineering buddies complain that he never comes to see them, but he says that they have a 50-50 chance. However, he had hung out with them just twice in the last 20 working days. Explain how this could happen, without resorting to random variation or a rare event (e.g., the uptown trains all broke down for 18 days).

   Extra Credit: *(1 point)* What city does Cory live in for this problem?

3. 10 cards are drawn from a standard 52-card deck. What is the probability that

   (a) *(4 points)* All 10 cards are of the same suit?
   (b) *(4 points)* All 10 cards are different suits?
   (c) *(4 points)* We have three (different) triples and a (different) singleton?
   (d) *(4 points)* We have a sequence of just 8 numbers (i.e., it's not part of a sequence of 9 or 10 numbers)?
   (e) *(4 points)* We have a sequence of just 8 numbers (i.e., it's not part of a sequence of 9 or 10 numbers), all of the same suit?

4. *(5 points)* One card is missing from a pack of 52 cards. From the remaining 51 cards five are drawn at random, and all are clubs. What is the probability that the missing card is a club?

5. (a) *(6 points)* A pair of dice is thrown n times in succession. What is the probability of throwing a double 6 at least once?

   (b) What are the following probabilities, and which is more likely?:

      i. *(3 points)* Getting at least 1 six when 6 dice are rolled?

      ii. *(3 points)* Getting at least 2 sixes when 12 dice are rolled?

      iii. *(3 points)* Getting at least 3 sixes when 18 dice are rolled?

   (c) **Extra Credit:** *(4 points)* Each of the above two problems is historical, involving famous mathematicians. For each one, name the mathematician(s) involved, the context (i.e., where the problem came up), and why each problem is historically interesting.

6. The pdf of X, the lifetime of a new Apple iPhone 3GS (measured in years), is given by

$$\frac{3}{2}\sqrt{x}e^{-\sqrt{x^3}}, \quad x > 0. \tag{1}$$

   (a) *(4 points)* Find $\mathrm{P}(X > 1)$.

   (b) *(4 points)* What is the cumulative distribution of X?

   (c) *(6 points)* Assuming the reliability of each iPhone is independent of any other one and that the lifetime is always modeled by (1) above, suppose that I buy an iPhone and replace it every year. What is the probability that the n^{th} iPhone is the first one to break during the year that I have it? That is, the first $n-1$ iPhones last during the year that I own it, but the n^{th} one breaks during the year that I own it?

7. Consider the experiment of tossing two fair tetrahedra (regular four-sided polyhedron), each with sides labeled 1-4. Let X denote the number on the downturned face of the first tetrahedron, and Y denote the larger of the two downturned numbers.

   (a) *(6 points)* Find the joint pmf of X and Y.

   (b) *(6 points)* Find the marginal pmfs for X and Y.

   (c) *(3 points)* Find $F_X(3)$.

   (d) *(3 points)* Find $F_{X,Y}(3, 3)$.

   (e) *(3 points)* Find $F_{X,Y}(-3, 3)$.

8. An urn contains 44 balls – one for each student of STAT/MATH 394. Each one contains the name of someone in Nate's STAT/MATH 394 class, and assume that names are distinct (i.e., no two students have the same name). Suppose that Nick, Cory, Zeinab and Neala are considered lucky names – meaning, for every one of those names that are picked, 5 points gets added to everyone's midterm score. A sample of size 10 is drawn from the urn with replacement. What is the probability that the sample will contain

   (a) *(4 points)* exactly 1 lucky name?

   (b) *(4 points)* at least 1 lucky name?

   (c) *(4 points)* exactly 4 lucky names?

9. An urn contains 44 balls – one for each student of STAT/MATH 394. Each one contains the name of someone in Nate's STAT/MATH 394 class, and assume that names are distinct (i.e., no two students have the same name). Suppose that Nick, Cory, Zeinab and Neala are considered lucky names – meaning, for every one of those names that are picked, 5 points gets added to everyone's midterm score. A sample of size 10 is drawn from the urn without replacement. What is the probability that the sample will contain

   (a) *(4 points)* exactly 1 lucky name?

   (b) *(4 points)* at least 1 lucky name?

   (c) *(4 points)* exactly 4 lucky names?

10. **Extra Credit:** A coin is tossed n times. What is the probability that heads will appear an even number of times? Specifically,

    (a) *(1 point)* Intuitively, what do you *think* the answer is, without doing any math?

    (b) *(4 points)* Now prove your intuitive answer in part (a). Is there a mathematical identity you are using in your proof?

    (c) *(5 points)* Now prove the mathematical identity you used in part (b).

    If somehow you (correctly) do part (b) *without* using a complicated mathematical identity, you get the full 9 points of parts (b) and (c).