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Name

1. Let  $X$  have the pdf

$$f_X(x) = \begin{cases} -\frac{x}{9} & x \in [-3, 0] \\ \frac{x}{4} & x \in (0, 2] \\ 0 & \text{otherwise} \end{cases} .$$

(a) (10 points) Find the pdf for  $Y = X^2$ .

(b) (10 points) Find the pdf for  $Y = X^3$ .

2. Tyler has taken the world's first photograph of a vacuum manifold, and people from around the world are coming to his chemistry lab to see it according to a Poisson process at a rate of 30 per hour.

(a) (5 points) What fraction of time intervals between successive arrivals will be longer than 2 minutes?

(b) (5 points) What is the probability that 5 or more people will arrive within 5 minutes?

3. The lifetime in hours of a radio tube of a certain type follows an exponential distribution with a mean of 1000 hours (i.e.,  $\lambda = \frac{1}{1000 \text{ hours}}$ ). A company producing these tubes wishes to guarantee them a certain lifetime.
- (a) (5 points) For how many hours should the tube be guaranteed to function in order to achieve a probability of 95% that it will function at least the number of hours guaranteed?

- (b) (10 points) An item is called *acceptable* if its lifetime runs longer than 50 hours. If an item is known to be acceptable, what is the probability it will run another 150 hours?

(c) (10 points) A batch of 10 items will be rejected if it contains at least one defective item. What is the expected number of batches which will be rejected in a shipment of 50 batches?

(d) (0 points) Would you say that this company has a quality control problem?

4. (5 points) A psychologist's measuring instrument produces a response  $Y$  that is distributed as normal  $(500, \sigma^2 = 100^2)$  for a certain population of students. Find the probability that the measurement  $Y$  is less than or equal to 600.

5. (15 points) A group consists of 10 college students: 4 freshmen, 3 sophomores, 2 juniors, and 1 senior. A committee of size 3 is randomly selected from this group. What is the probability that the committee will consist of students from three different classes?

6. A contractor has found through past experience that the low bid  $U$  for a job (excluding his own bid) is a random variable that is uniformly distributed over the interval  $(\frac{3}{4}c, 2c)$ , where  $c$  is a constant representing the contractor's cost (no profit or loss) of the job.

(a) (15 points) If the contractor does not get the job (his bid is greater than the low bid), the profit is defined as 0. If the contractor gets the job (his bid is the lowest), the profit is defined as the difference between his bid and his cost  $c$ . What should the contractor bid (in terms of  $c$ ) in order to maximize his expected profit?

(b) (10 points) Find the pdf of the random variable  $V = \frac{4}{5c}U - \frac{3}{5}$ .